

# $Z_N$ wall junctions: Monopole fossils in hot QCD

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## Abstract

We point out that the effective action of hot Yang–Mills theories has semi-classical solutions, which are naturally identified with monopole world lines, “frozen” into the short imaginary time dimension. The solutions look like wall junctions: lines along which  $N$  electric  $Z_N$  domain walls come together. They are instrumental in reconciling explicit perturbative calculations at high temperature with the magnetic  $Z_N$  symmetry.

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The idea that confinement in non abelian gauge theories is brought about by condensation of magnetic monopoles is an old one [1]. The monopole condensation has been shown to cause confinement in abelian - like models, such as the (2+1)- dimensional Georgi–Glashow model [2], or perturbed  $N = 2$  supersymmetric gauge theory in 3+1 dimensions [3]. However in a genuine non abelian context the monopoles have been rather elusive. Their identification through various abelian projections suffers from lack of gauge invariance and thus gauge dependence of various monopole properties [4].

Things would be clear if the monopoles were to appear as classical solutions of the QCD equations of motion. This, however, does not happen. This is not surprising by itself, since even if the monopoles do exist, they are strongly coupled and their density in the vacuum is not expected to be low. Thus it is hard to expect that they appear as well formed classical solutions.

The situation is somewhat different at high temperature. There QCD is perturbative, or at least has a perturbative sector which is usually described in terms of the weakly coupled effective theory for the Polyakov loop. This effective theory, being weakly coupled, may sustain meaningful classical solutions. The purpose of this note is to point out that such classical solutions exist, and that in a certain sense (to be explained below) they represent the “world lines” of the dynamical QCD monopoles. These word-lines are space-like and are squeezed in the short imaginary time direction.

Consider the high - temperature effective action of pure Yang–Mills theory in 3+1 dimensions. It is written in terms of  $N - 1$  independent phase fields  $A_\alpha$  — the phases of the eigenvalues of the Polyakov loop (see for example [5]). To one loop this effective action is

$$S = \frac{1}{2g^2N} \sum_{ij} (\partial_\mu A_{\alpha\beta})^2 + \frac{2}{3} T^4 \pi^2 \sum_{ij} B_4(A_{\alpha\beta}). \quad (1)$$

Here  $A_{\alpha\beta} = \frac{A_\alpha - A_\beta}{2\pi T}$ , with  $\alpha, \beta = 1, \dots, N$ , and  $A_N = -\sum_{\alpha=1}^{N-1} A_\alpha$ . The potential  $B_4$  is the Bernoulli polynomial. Since  $A_i$  are phases, the first homotopy group of the field manifold

is non-trivial, and thus there must be stable classical configurations with nontrivial winding. Let us require that all the phases  $A_\alpha$ ,  $\alpha = 1, \dots, N - 1$ , wind once when going around some straight line  $C$ . If we disregard the potential term in eq. (1), the equations of motion obviously have a solution with this boundary condition

$$A_\alpha(x) = \theta(x_{\text{perp}}), \quad (2)$$

where  $x_\perp$  are the coordinates perpendicular to  $C$ . The action per unit length of this solution is logarithmically divergent. Reintroducing the potential in the action, we see that the solution changes significantly. Now that the fields  $A_\alpha$  are massive it is energetically favourable to concentrate the winding within a two-dimensional surface of finite width, rather than have it delocalized in the whole space. The solution therefore will have a wall-like structure, where the fields  $A_\alpha$  vary within a width  $M_D^{-1}$  of half a plane with boundary  $C$ . In fact the finer structure of the solution can be understood quite easily. The potential in eq. (1) has  $N$  minima,  $A_\alpha = 2\pi n/N$ , for  $n = 0, 1, \dots, N - 1$ . While varying within the wall, the fields  $A_\alpha$  have to pass through all these minima. Since the interaction of  $A_\alpha$  is weak (especially at large  $N$ ), the wall will split into  $N - 1$  vacuum regions separated by the standard  $Z_N$  domain walls [6]<sup>2</sup>. This  $N$ -layered “sandwich” solution is depicted in Fig. 1. One can think of it as a wall junction, where  $N$  domain walls come together at the line  $C$ .

The action per unit length is now linearly divergent

$$S_{mf} = N\tilde{\sigma}L \quad , \quad (3)$$

where  $\tilde{\sigma}$  is the  $Z_N$  domain wall tension.

$$\tilde{\sigma} = \frac{4\pi^2 T^2 (N - 1)}{3\sqrt{3}g^2(T)N} \quad . \quad (4)$$

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<sup>2</sup>Of course if the boundary  $C$  is one straight line, the walls will repel each other and will spread like a fan. We have in mind however that  $C$  is closed, albeit its curvature is very small on the scale of  $M_D^{-1}$ .

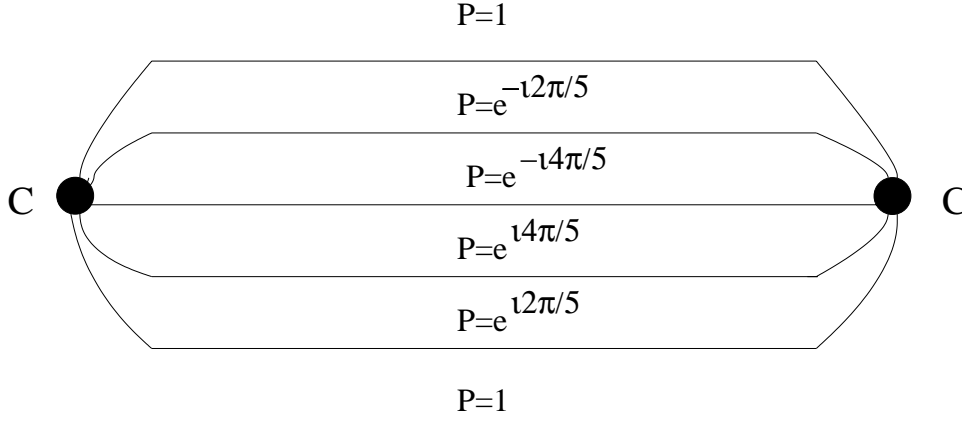


Figure 1: The classical monopole fossil configuration — the junction of  $N$   $Z_N$  domain walls (here  $N = 5$ ). The values of the trace of the Polyakov loop in each of the vacua in the “sandwich” are indicated. The contour  $C$  is perpendicular to the plane of the figure.

If instead of the straight line we choose a closed curve for  $C$ , the sandwich will be finite and will span the minimal area subtended by  $C$ .

What is the physical meaning of this solution? Imagine for a second that at zero temperature there are indeed monopoles. The charge of such a monopole would be  $N$  units of the fundamental magnetic charge allowed by the Dirac quantization conditions. Consider a world line of such a monopole with the unit tangential vector parallel to some coordinate axis  $x_\lambda$ . In the directions perpendicular to the world line, the monopole configuration has the Coulomb magnetic field  $\tilde{F}_{\lambda\nu} = \frac{N}{g} \frac{x_\nu}{x^3}$ ,  $\nu \neq \lambda$ . The action of such a configuration is proportional to the length of the world line, with the proportionality coefficient equal to the Coulomb energy of the monopole. Now let us increase the temperature. In the imaginary time formalism this amounts to making one direction finite with the size  $\beta$ . Let us take this finite direction to be perpendicular to  $x_\lambda$ . Because of the periodic boundary conditions in this finite direction, the magnetic flux cannot penetrate the boundary. The magnetic field lines will therefore bend as they come close to the boundary. Thus at distances larger than  $\beta$  the whole magnetic flux will be ef-

fectively squeezed into two transverse directions. At these distances the field will be two-dimensional Coulomb, rather than three-dimensional Coulomb, and the action density per unit length will logarithmically diverge with the volume of the system. This is entirely analogous to the change of the profile and the interactions of the instantons in (2+1)-dimensional Georgi–Glashow model [7, 8]. Also, since the compact dimension is the imaginary time, the components of the dual field strength which do not vanish in this configurations are the ones perpendicular to the time axis. Thus those are really electric fields rather than magnetic fields, and are representable in terms of the scalar potential  $A_0$ . Remembering that our fields  $A_\alpha$  are indeed the scalar potentials, we expect these configurations to appear directly in our effective action. Indeed, those are precisely the classical solutions described above. Indeed, disregarding the Debye mass term in the effective Lagrangian, the action per unit length of our solution is precisely the two-dimensional Coulomb energy of a monopole with magnitude  $\frac{N}{g}$ . The presence of the Debye mass above the deconfining transition is the reflection of the restoration of the magnetic  $Z_N$  symmetry [9], and it naturally affects the action of classical solutions. Again the situation is extremely similar to the case of (2+1) dimensions [7, 8]. The squeezed instantons interact logarithmically at low (but non-zero) temperature, but the interaction becomes linear above the deconfining transition.

Our classical solutions are therefore just the (space-like) worldliness of magnetic monopoles squeezed and preserved in the compact imaginary time dimension. Hence we will refer to these solutions as monopole fossils.

Since the world lines are space-like, these objects strictly speaking do not represent physical monopoles, but rather are related to magnetic vortices. To see this explicitly, let us consider the calculation of the expectation value of the  $N$ -fold 't Hooft loop [10], - the operator that creates a magnetic vortex with flux  $2\pi N/g$ :

$$V_N(C) = \exp \left\{ \frac{2\pi i}{g} \int_S d^2 S^i \text{Tr} Y E^i \right\} \quad (5)$$

where the hypercharge generator  $Y$  is defined as

$$Y = \text{diag}(1, 1, \dots, -(N-1)) \quad (6)$$

and the integration goes over a surface  $S$  bounded by the curve  $C$ . The operator does not depend on which surface  $S$  is chosen as long as its boundary is fixed [9, 11]. This operator is the operator of a singular gauge transformation in the hypercharge direction. The nature of the singularity is such that the gauge phase winds once when encircling the contour  $C$ . The path integral representation for this expectation value is then precisely the same as for the vacuum partition function except for the “boundary condition” imposed on  $A_\alpha$  in the integration domain: it must have a unit winding relative to  $C$ . Thus the steepest descent calculation of this expectation value is dominated by the classical solution we have just discussed.

The monopole fossils are instanton-like objects, which describe the process of creation of a magnetic vortex with magnetic flux  $\frac{2\pi}{g}$ . To avoid confusion we wish to make the following comment. The operator  $V_N$  defined in eq. (5) is in fact a trivial operator. Non-perturbatively, it is equivalent to the unit operator. This is simplest to understand by noting that it commutes with all gauge-invariant operators [9]. The reason this is not obvious in our derivation is that we implicitly assumed that the integration in the thermal path integral is perturbative and is thus only over the small fields  $A_\alpha$ .

This is not to say that the monopole fossils are not important. Their importance is precisely in restoring the triviality of the  $N$ -fold vortex operator within the perturbative/semiclassical domain. In this they are directly analogous to usual instantons in QCD. Recall that if we neglect instantons in the QCD vacuum path integral, the operator of a large gauge transformation  $U$  is non-trivial. For example its insertion into the path integral changes the boundary conditions on the gauge fields by changing the total topological charge by one unit. Thus perturbatively the calculation of  $\langle U \rangle$  is dominated by the one instanton configuration and the result would be  $\exp\{-S_{inst}\}$ . It is only after

we sum over all “dynamical” instantons and anti-instantons in the path integral that the triviality of  $U$  is restored. Technically this is because the vev of  $U$  will be dominated by a configuration where a “dynamical” anti-instanton from the vacuum ensemble will sit on top of the instanton induced by the explicit insertion of  $U$ . Thus the leading contribution will be 1 rather than  $\exp\{-S_{inst}\}$ . More generally, in calculation of any correlation function insertions of  $U$  are unimportant. Such an insertion amounts to adding one more instanton to the ensemble which, as it is, has an indeterminate number of randomly distributed instantons and anti-instantons. Thus the one extra insertion does not change the ensemble, and this restores the triviality of the operator  $U$  in such a semi-classical context.

Exactly the same thing happens with our monopole fossils. The perturbative result  $\langle V_N(C) \rangle = \exp\{-S_{mf}\}$  gets changed to  $\langle V_N \rangle = 1$  once we sum over all possible monopole fossils in the thermal ensemble, since there is always a “dynamical” fossil to screen the one induced by the insertion of  $V$ . Note that the action of a large monopole fossil is very large (proportional to the area with large tension), and thus the fossils are not abundant in the vacuum ensemble. This does not prevent them from dominating the path integral for  $\langle V_N \rangle$ , just as the instantons with large action dominate  $\langle U \rangle$ .

Another quantity in which the contributions of fossils are important are expectation values of  $k$ -fold 't Hooft loops with  $k < N$ . The calculation of these expectation values has been performed recently in [12] with the result

$$\langle V_k(C) \rangle = \exp\{-\tilde{\sigma}_k S\} \tag{7}$$

with

$$\tilde{\sigma}_k = \frac{N-k}{N-1} k \tilde{\sigma} \quad . \tag{8}$$

The main feature of this result which is of interest to us, is that  $\langle V_k(C) \rangle = \langle V_{N-k}(C) \rangle$ . This is of course what one expects non-perturbatively. For large areas,  $\langle V_k(C) \rangle$  measures

the longest correlation length in the channel with magnetic flux  $2\pi k/g$ . Since the global magnetic symmetry in the theory is  $Z_N$ , it must be true that the same correlation length also appears in the channel with flux  $2\pi(k - N)/g$ . On the other hand if the symmetry were  $U(1)$  no such relation would hold. Thus eq. (8) is consistent with the magnetic symmetry being  $Z_N$  rather than  $U(1)$ .

However, from the point of view of a purely perturbative calculation, this result seems surprising. The interaction between two  $k = 1$  walls is only via the phase fields  $A_\alpha$ . Such an interaction is usually repulsive. In fact one can readily convince oneself that at least when the separation between the walls is larger than the inverse Debye mass, the interaction is indeed repulsive. Thus, if we were to look for a stable configuration with  $k = 2$  by just solving the classical equations with the boundary conditions corresponding to  $k = 2$ , we would find that the stable solution is two walls fairly well separated in the transverse direction with the action  $2\tilde{\sigma}$ . And the same would happen for any  $k$  - the classical action will be just proportional to  $k$ . The authors of [12] note explicitly that if they use the simple definition of the  $k$ -fold 't Hooft loop and perform straightforward perturbative calculation they indeed get this result. <sup>3</sup>

This puzzle is resolved by the presence of the monopole fossil solutions. Without their contributions, the perturbation theory distinguishes sharply between  $V_k$  and  $V_{N-k}$ , and thus leads to different values for  $\tilde{\sigma}_k$  and  $\tilde{\sigma}_{N-k}$  string tensions. On the other hand if we do take them into account it is easy to see that when calculating  $\langle V_{N-k}(C) \rangle$  a single anti-fossil sitting right on top of the contour  $C$  will turn  $\langle V_{N-k}(C) \rangle$  into  $\langle V_{-k}(C) \rangle$  (which by charge conjugation is equal to  $\langle V_k(C) \rangle$ ). The fossil does not have to sit right on top of  $C$  to give a significant contribution. Its position and shape should be integrated over. The mixing between  $\langle V_{N-k}(C) \rangle$  and  $\langle V_{-k}(C) \rangle$  that the fossils bring about must also reduce the lowest mass in this channel, and thereby bring the perturbative result

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<sup>3</sup>The actual calculation in [12] is performed differently, by globally sampling the space of allowed configurations of  $A_\alpha$  and thus avoiding this problem.



for  $\tilde{\sigma}_k = k\tilde{\sigma}$  in agreement with eq. (8). We do not attempt the explicit calculation of this effect here, but note that in weakly interacting theories in 2+1 dimensions a similar calculation has been performed in detail in [8]. It is indeed shown there that the high-temperature remnants of monopole-instantons have the effect of increasing the longest correlation length in a channel with a given magnetic flux through precisely the same mixing mechanism as discussed above.

Finally we note that other classical configurations with nontrivial winding can be considered. For example, one can require that  $A_1$  winds around some contour, but the other  $A_\alpha$ ,  $\alpha = 2, \dots, N-1$ , do not. This solution also has the meaning of a monopole fossil, but this time of a monopole which carries the flux only within a particular  $U(1)$  subgroup. These configurations may also be important in semi-classical calculations of the type described above.

To conclude, we pointed out in this note that the effective action of high-temperature Yang–Mills theory has classical solutions that are naturally identified with the space-like world lines of magnetic monopoles squeezed in the imaginary time direction. The action of these solutions is proportional to the minimal area subtended by the world-line  $C$ . Such a solution looks like a junction of  $N$   $Z_N$  domain walls terminating on  $C$ . Their importance in the semi-classical context is to break the magnetic symmetry down to  $Z_N$  from the apparent  $U(1)$  by mixing the correlation functions of  $V_k(C)$  and  $V_{k-N}(C)$ .

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